Calculus III (SM221, SM221S, SM221Y) FINAL EXAM

You should have a calculator. Write your name, alpha number, and section on both the blue book(s) and the bubble sheet. Bubble in your alpha number in the left-most columns on the bubble sheet.

Part One. Multiple choice (50%). The first 20 problems are multiple choice. Fill in the letter of the best answer on the bubble sheet. There is no penalty for a wrong answer. YOU MUST ALSO WRITE THE ANSWER AND SHOW ALL YOUR WORK IN YOUR BLUE BOOK.

---

1. The range of the function given by $g(x,y) = \sqrt{1-x^2-y^2}$ is

a. all real numbers b. the interval $[0,\infty)$ c. the interval $[0,1]$
d. the interval $(-\infty,1]$ e. all pairs (x,y) with $x^2+y^2 \le 1$.

---

2. The point with cylindrical coordinates $(r,\theta,z) = (2,\pi/3,5)$ is at what distance from the origin?

a. 1 b. 2 c. 3 d. 5 e. $\sqrt{29}$

---

3. Which best describes the surface given in spherical coordinates by $\phi = \pi/3$?

a. a sphere b. a single cone c. a cylinder
d. a full vertical plane e. part of a vertical plane

---

4. The Maple command `spacecurve([sin(t),t,cos(t)],t=0..4*Pi);` will give which result below? (Axes included, properly labeled, and output rotated to more standard position)

a. b. c. d. e.

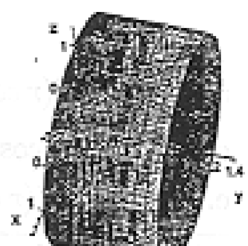
---

5. If $\mathbf{r}'(t) = \langle e^t, \cos(t), t^2\rangle$ and $\mathbf{r}(0) = \langle 2,3,-1\rangle$, then $\mathbf{r}(t)$ equals which of the following?

a. $\langle e^t, -\sin(t), 2t\rangle$ b. $\langle e^t, \sin(t), t^3/3\rangle$ c. $\langle 1+e^t, 3+\sin(t), -1+t^3/3\rangle$
d. $\langle e^t+2, -\sin(t)+3, 2t\rangle$ e. $\langle 1+e^t, 2+\cos(t), -1+t^2\rangle$

---

6. The graph of the cylinder with y-axis for an axis can be drawn as shown with which of the following Maple commands? (Rotated to standard position)



```
a. plot3d([x,cos(t),sin(t)], x=0..1, t=0..2*Pi);
b. plot3d([cos(t),y,sin(t)], y=0..1, t=0..2*Pi);
c. plot3d([cos(t),sin(t),z], z=0..1, t=0..2*Pi);
d. plot3d([t,cos(t),sin(t)], t=0..2*Pi);
e. plot3d([cos(t),t,sin(t)], t=0..2*Pi);
```

7. Suppose wave heights, $f(v,t)$, as a function of wind speed v and duration t are given by the following chart.

$v \setminus t$	5	10	15
10	2	3	3
20	8	8	9
30	12	14	15

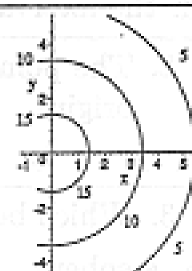
Which of the following is the best estimate for $f_v(20,10)$?

a. 0.05 b. 0.55 c. 1 d. 2 e. 20

8. Suppose $z = xy^2 + \ln(x)$, $\frac{dx}{dt} = e^{(t^2)}$, $\frac{dy}{dt} = \sqrt{t+4}$, and when $t=0$: $x=3$, $y=5$. Which of the following is closest to $\frac{dz}{dt}$ when $t=0$?

a. 79 b. 81 c. 83 d. 85 e. 87

9. The Marines are on a small hill with the given contour map describing the height function $h(x,y)$. For $\mathbf{u} = \langle 0.6, -0.8 \rangle$, the directional derivative $(D_{\mathbf{u}}h)(1,-1)$ is closest to:



a. -2.5 b. -1 c. 0 d. 1 e. 2.5

10. A pile of sand within a 20 ft by 40 ft walled rectangle has depths in feet given by the accompanying chart at various locations. Using the midpoint rule with 4 subrectangles (dividing by 2 in each of the x and y directions) the approximation to the volume of sand is:

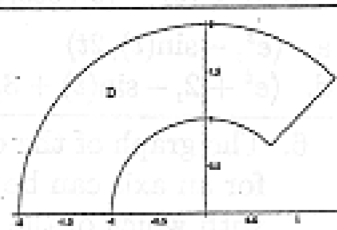
$x \setminus y$	0	10	20	30	40
0	2	6	3	5	4
5	1	7	6	8	5
10	3	5	4	6	2
15	5	6	7	9	3
20	1	4	3	10	6

a. $300\,ft^3$ b. $600\,ft^3$ c. $900\,ft^3$ d. $1200\,ft^3$ e. $1500\,ft^3$

11. If $\int_0^2 f(x,y)\,dy = x - 2x^2$ and $R = [0,1] \times [0,2]$ (the rectangle with $0 \le x \le 1$ and $0 \le y \le 2$), then $\iint_R f(x,y)\,dA =$

a. -3 b. $-1/6$ c. 0 d. $1/6$ e. 3

12. Which iterated integral equals $\iint_D f(x,y)\,dA$ where D is as drawn?



a. $\int_{\pi/4}^{\pi} \int_1^2 f(r\cos(\theta), r\sin(\theta))\, r\, dr\, d\theta$

b. $\int_{\pi/4}^{\pi} \int_0^2 f(r\cos(\theta), r\sin(\theta))\, r\, dr\, d\theta$

c. $\int_0^{\pi} \int_1^2 f(r\cos(\theta), r\sin(\theta))\, r\, dr\, d\theta$

d. $\int_0^{-\pi/4} \int_1^2 f(r\cos(\theta), r\sin(\theta))\, r\, dr\, d\theta$

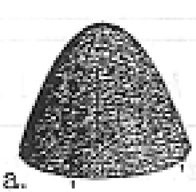
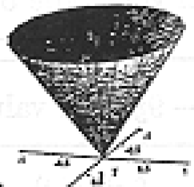
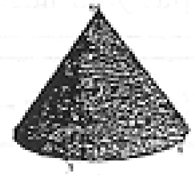
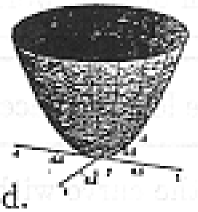
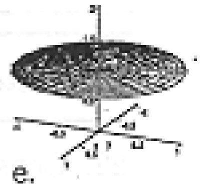
e. $\int_{\pi/4}^{\pi} \int_{-2}^{-1} f(r\cos(\theta), r\sin(\theta))\, r\, dr\, d\theta$

13. The surface area of the part of the graph of $f(x,y)$ over the region R in the xy-plane can be found by evaluating $\iint_R g(x,y)\,dA$ where $g(x,y)$ is given by

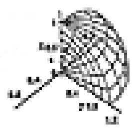
a. $\sqrt{1 + f_x(x,y)^2 + f_y(x,y)^2}$ b. $\sqrt{1 + f_x(x,y) + f_y(x,y)}$ c. $1 + f_x(x,y) + f_y(x,y)$

d. $1 + f_x(x,y)^2 + f_y(x,y)^2$ e. $f(x,y)$

14. $\int_0^{2\pi}\int_0^1\int_0^{r^2} r\,dz\,dr\,d\theta$ gives the volume under which surface drawn below (each has height 1)?

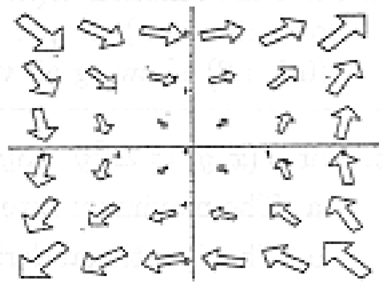

a.

b.

c.

d.

e.

15. Which of the following will give the mass of the 1/8 ball of radius 1 as drawn, assuming density equals distance from the origin?



a. $\int_0^{\pi/2}\int_{\pi/2}^{\pi}\int_0^1 \rho^3\sin(\phi)\,d\rho\,d\theta\,d\phi$ b. $\int_0^{\pi}\int_{\pi/2}^{\pi}\int_0^1 \rho^3\sin(\phi)\,d\rho\,d\theta\,d\phi$
c. $\int_{\pi/2}^{\pi}\int_0^{\pi}\int_0^1 \rho^3\sin(\phi)\,d\rho\,d\theta\,d\phi$ d. $\int_0^{\pi}\int_0^{\pi}\int_0^1 \rho^3\sin(\phi)\,d\rho\,d\theta\,d\phi$
e. $\int_0^{\pi/4}\int_{\pi/4}^{\pi/2}\int_0^1 \rho^3\sin(\phi)\,d\rho\,d\theta\,d\phi$

16. Which vector field below is plotted?
(vectors are all scaled equally)



a. $\langle y, x\rangle$ b. $\langle y, y\rangle$ c. $\langle x, x\rangle$
d. $\langle x, y\rangle$ e. $\langle -x, -y\rangle$

17. Evaluate $\int_C x\,dy$ where C is given by $x = 3t$, $y = t^2$, $0 \le t \le 3$.

a. 0 b. 9/2 c. 27/2 d. 54 e. 144

18. Suppose f is a potential function for $\mathbf{F}$ (so that $\nabla f = \mathbf{F}$). This table gives certain values of $f(x, y)$. Use it to find $\int_C \mathbf{F}\cdot dr$ where C is parameterized by $x = 2 - 2t$, $y = t^3$, $0 \le t \le 1$.

x\y	0	1	2
0	3	4	5
1	5	8	11
2	9	16	20

a. -17 b. -5 c. 0 d. 4 e. 8

19. By Green's Theorem, which of the following will be the area enclosed by a positively oriented smooth simple closed curve C?

a. $\oint_C y\,dx + x\,dy$ b. $\oint_C 2y\,dx + x\,dy$ c. $\oint_C y\,dx + 2x\,dy$
d. $\oint_C 2y\,dx + 2x\,dy$ e. $\oint_C x\,dx + y\,dy$

20. If $\mathbf{F}(x, y, z)$ has a constant divergence 3 ($\operatorname{div}\mathbf{F} = 3$) over the solid cube with vertices at $(\pm1, \pm1, \pm1)$, then for C the boundary of that cube, $\iint_C \mathbf{F}\cdot d\mathbf{S}$ equals:

a. 0 b. 5 c. 15 d. 21 e. 24

Part Two. Longer Answers (50%). The remaining 10 problems are not multiple choice. Again, show all of your work and put your answers in your blue book(s).

21. Sketch the level surfaces of $f(x,y,z) = x - 4z^2 - 9y^2$ for values of $k = -1, 0, 1$.

22. Consider the curve with parametric equations $x = 2\sin(3t)$, $y = 5t$, $z = 2\cos(3t)$.
   a. Find the length of the curve from $t = 0$ to $t = \pi$.
   b. Find parametric equations for the tangent line to the curve at the point $(-2, 5\pi/2, 0)$.

23. Does a football accurately kicked at a $60°$ angle with the horizontal at an initial speed of 41 ft/sec clear a 10 ft high crossbar 40 ft away? Find the position $\mathbf{r}(t)$ at any time t and show all work.

24. Find the function $L(x,y)$ that is the linear approximation (also called the tangent plane approximation) to $g(x,y) = \sin(x) + e^y$ at $(0,1)$ and use it to give an approximation of $g(0.1, 1.2)$ showing all work.

25. For $f(x,y) = 2x^2y - 3y^2$ and $P = (1,2)$ find
   a. The maximum rate of change of f at P and the direction in which it occurs.
   b. The directional derivative of f at P in the direction of $\mathbf{i} + 2\mathbf{j}$.

26. Consider the doubly iterated integral given in Maple by

```
int(int(exp(x^2),x=2*y..4),y=0..2);
```

Draw the region of integration and give the Maple command to evaluate the integral over the same region but with the order of integration reversed.

27. Define the curl by giving a formula for it for all functions $\mathbf{F}(x,y,z)$. Use this definition to show that for vector valued functions of the form $\mathbf{F}(x,y,z) = \langle f(x), g(y), h(z)\rangle$ we have $\text{curl}\,\mathbf{F} = 0$.

28. Evaluate the surface integral $\iint_S (x^2 + y^2)\, dS$ where S is the helicoid with vector equation $\mathbf{r}(u,v) = u\cos(v)\mathbf{i} + u\sin(v)\mathbf{j} + v\mathbf{k}$, $0 \le u \le 1$, $0 \le v \le \pi$.

29. Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x,y,z) = x\mathbf{i} + x^2\mathbf{j} + yz\mathbf{k}$ and C is the curve of intersection (traversed counterclockwise when viewed from above) of the plane and cylinder given by $2x + z = 3$ and $x^2 + y^2 = 4$.

30. Use the Divergence Theorem to calculate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x,y,z) = 3xy\mathbf{i} + x^2z\mathbf{j} - z^2\mathbf{k}$ and S is the surface of the tetrahedron with vertices $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$.

(end)